Fall 2019 TAGMaC List of Abstracts

10:00–11:00 Welcome and Keynote Talk (SAS 2203)

10:00 *Bad Behavior*
Lillian Pierce pierce@math.duke.edu
What do you do with a person who behaves in the worst possible way at every point in time? Well, I don't know. But if you ask instead about a mathematical operator that picks out the worst possible behavior of a function at every point in time, we sometimes know how to control it. In fact, such operators are very useful in harmonic analysis, and have been studied in various guises over the past century. We will survey several classical operators of this kind, and then will explore a few modern questions, which are tantalizingly beautiful but appear to be very difficult.

11:00–12:45 Session I

SAS 2225: Biomathematics

11:00 *Analyzing the Structural Propertiesof Pulmonary Arterial Networks*
Megan Chambers
NCSU advisor: Mette Olufsen mjchambe@ncsu.edu
From micro-CT images of the lungs of mice, one can observe that the pulmonary arterial network forms a tree-like structure. In collaboration with Kitware, Inc. we have been able to use 3D Slicer, an open source image analysis software, to extract the representative graph structures from these images. These graphs include the (x,y,z) coordinates of terminal and bifurcation nodes and edge points, as well as the vessel radii at these points. While it is apparent in these images that the vessels form a branching tree, the exact topological and geometric structure of the networks varies widely due to experimental conditions and parameters set during the segmentation process. In this talk, we explore the various geometric and topological relationships that may hold in the vessel network. Our work is based on Olufsen et al. (2000), which hypothesizes that the arteries form a self-similar "structured tree", whose pattern is generated by quantifying certain geometric parameters of the vessels, such as scaling factors for radii and length to radius ratios. Moreover, Olufsen et al. assumes that the structured tree parameters remain constant throughout the network; in this talk we examine the validity of these hypotheses. We also aim to use the structured tree model parameters to distinguish between control and hypertensive networks.

11:35 *Coincidence of homeostasis and bifurcation in feedforward networks*
William Duncan, Martin Golubitsky
Duke advisor: Michael Reed `wduncan@math.duke.edu`
Homeostasis can be studied by restricting one's attention to homeostasis points—points at which a component of the dynamical system has a vanishing derivative with respect to an input parameter. Such homeostasis points play an important role in physiology and medicine. In a feedforward network with a dynamical system at each node, if a node has a homeostasis point, downstream nodes will inherit it. This is the case except when the downstream node has a bifurcation point coinciding with the homeostasis point. This talk will discuss this phenomena of homeostasis-bifurcation points. Near these points, the downstream node often exhibits complex behavior as the input parameter is varied. In the case of steady state bifurcation this includes switch-like behavior and multiple homeostatic plateaus. In the case of Hopf bifurcations the downstream node may have limit cycles with a wide range of near-constant amplitudes and periods.

12:10 *Estimation of Cell Division Generations Using Binary Trees*
Christine Mennicke
NCSU advisor: Mansoor Haider `cvmennic@ncsu.edu`
Progenitor cells in the developing brain divide and differentiate into functional cells like neurons or glial cells. The history of a cell's divisions can be represented as a binary tree, with a progenitor cell at the root and differentiated cells as its leaves. We use recursion to represent all possible binary trees with a particular number of leaves N and form the distribution of possible tree depths given N. From this distribution, we estimate the most likely rounds of cell division required to produce the glial cell counts from real data.

SAS 2235: Representation Theory and Lie Algebra

11:00 *Root Components and Tensor Product Decomposition for Affine Lie Algebras*
Sam Jeralds
UNC-CH advisor: Shrawan Kumar `sjj280@live.unc.edu`
In his proof of the Wahl Conjecture for flag varieties, Kumar exhibited certain irreducible "root components" in the tensor product of two highest-weight irreducible representations of a semisimple Lie algebra. In this talk, we will present recent results extending this construction to the case of affine Lie algebras.

11:35 *Twisted De Rham Complex on Line and $\mathfrak{sl}2$ Singular Vectors*

Alexey Slinkin, Alexander Varchenko

UNC-CH advisor: Alexander Varchenko `slinalex@live.unc.edu`

We consider two complexes. The first complex is the twisted De Rham complex of scalar meromorphic differential forms on projective line, holomorphic on the complement to a finite set of points. The second complex is the chain complex of the Lie algebra of $\mathfrak{sl}$-valued algebraic functions on the same complement, with coefficients in a tensor product of contragradient dual Verma modules over the affine Lie algebra $\mathfrak{sl}2$. In this talk, we will describe the monomorphism of the first complex to the second that was suggested by A. Varchenko and V. Schechtman. We will see that under this monomorphism the existence of singular vectors in the Verma modules (the Malikov-Feigin-Fuchs singular vectors) is reflected in the relations between the cohomology classes of the De Rham complex.

12:10 *Darboux Transformations and Fay Identities of the Extended Bigraded Toda Hierarchy*

Anila Yadavalli, Bojko Bakalov

NCSU advisor: Bojko Bakalov `ayadava@ncsu.edu`

Integrable hierarchies arise by starting with a differential equation that models a real life process and constructing a system of infinitely many differential equations that can be solved simultaneously. Some classical examples of integrable hierarchies are the KP, KdV, and Toda hierarchies. Solutions to such hierarchies can be studied algebraically using tools such as Darboux transformations. I will begin by introducing the extended bigraded Toda hierarchy (EBTH). Darboux transformations for the EBTH were determined by Li and Song in 2016. I will conclude by showing that the action of the Darboux transformation on solutions to the EBTH is given by a vertex operator, and that this result leads to Fay identities for the EBTH. This is joint work with B. Bakalov.

SAS 2229: Numerical Analysis and Inverse Problems

11:00 *Inferring the micro architecture of cortical bone from ultrasound attenuation*

Rebekah White

NCSU advisor: H.T. Banks `rdwhite@ncsu.edu`

The micro-architecture of cortical bone is affected by osteoporosis. However, employing ultrasonic waves in order to characterize this degradation has yet to be done. Here, we use a 3D physics based model, derived from the Independent Scattering Approximation (ISA), in order to model the frequency dependent ultrasonic attenuation in cortical bone. Furthermore, Bessel functions are used to estimate the forward scattering pressure of longitudinally propagating elastic waves. We generate attenuation data for 3D samples with both monodisperse and polydisperse pore diameters. We then formulate appropriate inverse problems, using the Prohorov Metric Framework (PMF) in the case of polydisperse pore sizes, in order to infer the optimal micro-architectural parameters. With this we show it is possible to quantify osteoporosis levels from simulated ultrasound data.

11:35 *Ensuring a sparse and binary A-optimal design for Bayesian linear inverse problems*

Elizabeth Herman, Alen Alexanderian and Arvind Saibaba

NCSU advisors: Alen Alexanderian and Arvind Saibaba `eaherman@ncsu.edu`

We tackle the problem of finding optimal sensor placements for large-scale Bayesian linear inverse problems. This can be formulated as an Optimal Experimental Design (OED) problem, which involves optimizing certain design criteria over the design variables. Specifically, we consider the A-optimal criterion, which involves the computation of the trace of the posterior covariance matrix. The optimization problem for finding optimal designs strikes a balance between minimizing the design criterion and the number of sensors deployed. To control the sparsity of the sensor placements, we use an approach based on reweighted $\ell 1$ minimization. We present illustrative numerical results for a model contaminant source identification problem, where the inverse problem seeks to recover the initial condition of a contaminant plume using discrete measurements of the contaminant in space and time.

12:10 *Numerical approach to the solution of the inverse source problems for the radiative transfer equation*

Alexey Smirnov, Michael V. Klibanov, Loc Nguyen

UNC-Charlotte advisor: Michael V. Klibanov `asmirno2@uncc.edu`

A new numerical method for an inverse source problem for the radiative transfer equation in the specific case of incomplete data is proposed. The quasi-reversibility method is applied to reduce the original inverse source problem for the integro-differential transfer equation to the minimization problem. The existence, uniqueness regularized solution of minimization problem was proved. A new discrete Carleman estimate was introduced to show the convergence of the regularized solution. Numerical simulations were performed demonstrating the potential of the proposed method in applications to the real-world X-ray tomography problems in multiple scattering regimes.

2:00–4:20 Session II

SAS 2225: PDEs and Harmonic Analysis

2:00 *Approximation of Correctors and Multipoles in Random Elliptic Media*

Lihan Wang, Jianfeng Lu, Felix Otto

Duke advisor: Jianfeng Lu `lihan@math.duke.edu`

We consider the whole-space decaying solution of the PDE $-\nabla \cdot a\nabla u = \nabla \cdot g$ in space dimension $d = 3$, where a is a realization of a stationary, uniformly elliptic, unit range ensemble of random field, and g is a deterministic right-hand-side compactly supported in a ball of size ℓ. Given the coefficient field in a large box of size L with $\ell \ll \frac{L}{\ln L}$, we are interested in an algorithm to compute ∇u with the "best" artificial boundary condition on the domain of size L which describes the correct long-range multipole behavior. We want to show that, with high probability, our algorithm reaches the CLT-type lower bound of error previously proven by Lu and Otto.

2:35 *The Unified Transform Method for linearized Korteweg–de Vries Equation*

Mick Stukes
UNC-Charlotte gstukes@uncc.edu

I will review a method of solving initial boundary value partial differential equations developed by A. S. Fokas between 1997 and 2008, in a manner that does not require the reader to have significant background in PDEs. The session will present the reader with all necessary preliminary material to understand the proof of the method. I will then present the method as it applies to linear evolution PDEs on the half-line and finite interval. Examples of both types are provided to illustrate the method in action.

SAS 2235: Number Theory, Combinatorics, and Algebra

2:00 *Graph Associahedra, Type A Root Systems (and beyond?)*

Jordan Almeter
NCSU advisor: Nathan Reading jgalmete@ncsu.edu

Graph Associahedra are well-known combinatorial objects, defined by relations between connected subgraphs, and which generalize associahedra and permutahedra. This talk will give an introduction to these objects in relation to the "Type A_n" root system, and suggest ideas related to my current work in generalizing graph associahedra to other root systems.

2:35 *Rational Points in Regular Orbits attached to Infinitesimal Symmetric Spaces*

Chung-Ru Lee, Trung Can, Benjamin Nativi, and Gary Zhou
Duke advisor: Jayce Getz chung.ru.lee@duke.edu

Motivated by problems arising in the relative trace formula and arithmetic invariant theory, we prove the existence of rational points on orbits in certain infinitesimal symmetric spaces. As an application, we prove analogous results for orbits in certain global reductive symmetric spaces.

3:10 *Infinite Full Twists in the Kuperberg Spider for sp4*

Onkar Gujral, Michael Abel and Dmitry Vagner
Duke advisor: Ezra Miller onkar.gujral@duke.edu

The category of fundamental representations of some semi-simple lie algebras via a braided monoidal category (with duals) have been associated with corresponding diagrammatic algebras. A braiding means that we get crossings in our algebra. Now rather than working in the abstract with these representations, we can instead play with this diagrammatic algebra. The semi-simple lie algebra of interest here is sp4. The strands in the diagrammatic algebra associated with sp4 are of 2 colors, labelled colors 1 and 2.

One may then ask what an infinite full twist of these crossings converges to. Previously, by the individual works of Rozansky, Rose, and some others, it has been shown that the infinite full twist in sln converges to a Jones-Wenzl projector, for $n = 2, 3$, and then eventually all n. The Jones-Wenzl projector is a special kind of non-zero idempotent element that is annihilated by all cup-caps and forks in the diagrammatic algebra. This result is known for the categorified version for sln.

Working with color-1 infinite full twists, we show that the same holds for sp4 in the decategorified version.

Mentors: Dr Michael Abel and Dmitry Vagner, as part of the PRUV program at Duke University.

3:45 *Magnitude Homology: Structure and Torsion*
Victor Summers
NCSU advisor: Radmila Sazdanovic `vwsummer@ncsu.edu`
Many mathematical constructions come equipped with a canonical measure of size; the cardinality of a set, Euler characteristic of a topological space, dimension of a vector space, to name just three. T. Leinster added magnitude of a metric space to the list of cardinality-like invariants. Graphs may be viewed as metric spaces with the shortest-path metric, and as such they have magnitude. R. Hepworth and S. Willerton went on to categorify the magnitude of graphs, realizing the power series invariant as the graded Euler characteristic of a bigraded homology theory; magnitude homology. In this talk I will begin by constructing magnitude homology and describing various properties of magnitude which lift to the level of homology. Then I will discuss some results on the existence and structure of torsion in magnitude homology.

SAS 2229: Analysis and Differential Geometry

2:00 *Fluid Mechanics (Title TBD)*
Hang Yang
Duke advisor: Alexander Kiselev `dylan.hangyang@gmail.com`
Abstract TBD

2:35 *Euclidean Signatures of Non-Congruent Planar Curves*
Eric Geiger
NCSU advisor: Irina Kogan `edgeiger@ncsu.edu`
Two planar curves congruent under a Euclidean transformation will aways share the same signature curve. However, the converse is not always true as displayed in constructions by Musso and Nicolodi using planar curves with “generalized vertices” or intervals of constant curvature . In this talk we will construct families of non-congruent planar curves of class C^3 without generalized vertices.

3:10 *How to project onto the intersection of cone and a sphere?*
Minh Bùi, Heinz H. Bauschke and Xianfu Wang
NCSU advisor: Patrick L. Combettes `mnbui@ncsu.edu`
Only in rare cases is it possible to obtain a "closed form" for the projector associated with the intersection of two sets—even when both sets are convex and their projectors have explicit descriptions. In this talk, we shall deal with the intersection of a cone and a sphere (centered at the origin) in a real Hilbert space. (Note that the latter is a nonconvex set.) Under a mild assumption on the cone, we obtain a formula describing the full (possibly set-valued) projector. Several examples are provided to illustrate our formula, e.g., when the cone is a subspace (the intersection is thus a circle), a finitely generated cone, the ice cream cone (aka Lorentz cone), or the cone of positive semidefinite matrices.